Problem 18

Because Earth's rotation is gradually slowing, the length of each day increases: The day at the end of 1.0 century is 1.0 ms longer than the day at the start of the century. In 20 centuries, what is the total of the daily increases in time?

Solution

Assume that the Earth's rotation is slowing at a linear rate. Then the rate at which the day increases is the following conversion factor.

$$\frac{1.0 \text{ ms}}{1.0 \text{ century}}$$

Since we want the total of the daily increases in time, change the units of the denominator to days. And to express the final answer most conveniently, change the units of the numerator to hours.

$$\frac{1.0 \text{ ms}}{1.0 \text{ century}} \times \frac{1 \text{ s}}{1000 \text{ ms}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ century}}{100 \text{ years}} \times \frac{1 \text{ year}}{365 \text{ days}} = \frac{1 \text{ hr}}{1.314 \times 10^{11} \text{ days}}$$

The daily increase for each of the first three days (in hours) is

$$\frac{1 \text{ hr}}{1.314 \times 10^{11} \text{ days}}(1 \text{ day}) \quad \text{and} \quad \frac{1 \text{ hr}}{1.314 \times 10^{11} \text{ days}}(2 \text{ days}) \quad \text{and} \quad \frac{1 \text{ hr}}{1.314 \times 10^{11} \text{ days}}(3 \text{ days}).$$

To obtain the sum of the daily increases after twenty centuries, use a sum from day 1 to day $20 \times 100 \times 365$ to add them all together.

$$\begin{aligned} \text{Total} &= \sum_{n=1}^{20\times100\times365} \frac{1}{1.314 \times 10^{11}}(n) = \frac{1}{1.314 \times 10^{11}} \sum_{n=1}^{730\,000} n = \frac{1}{1.314 \times 10^{11}} \left[\frac{1}{2}(730\,000)(730\,001)\right] \\ &\approx 2.0 \text{ hours} \end{aligned}$$